DOCUMENT RESUME

ED 309 969 SE 050 808

TITLE Kindergarten Mathematics.
INSTITUTION Ohio State Dept. of Education, Columbus. Div. of Inservice Education.
PUB DATE 88
NOTE 20p.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Early Childhood Education; *Fundamental Concepts; *Kindergarten; *Mathematical Concepts; *Mathematics Curriculum; *Mathematics Instruction; Mathematics Materials; Primary Education

ABSTRACT

This monograph is one of a series describing goals, student outcomes, and instructional techniques for mathematics in the elementary and middle grades. Sections included are: (1) "Introduction"; (2) "Overview of the Kindergarten Mathematics Program"; (3) "Kindergarten Teaching Considerations" (considering variability, learning characteristics, language development, manipulatives, and personalizing instruction); (4) "Teaching Emphases" (discussing prenumber experiences, number activities, counting, measurement, geometry, and problem solving with sample questions and activities); (5) "Major Learning Outcomes for Kindergarten"; (6) "Evaluation"; (7) A Final Word"; and (8) "Resources." (YP)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED309969

KINDERGARTEN MATHEMATICS

K

OHIO DEPARTMENT OF EDUCATION COLUMBUS•1988

This monograph is one in a series of similar documents describing goals, student outcomes, and instructional techniques for mathematics in the elementary and middle grades. There are eleven monographs in this series – one which is an overview of goals, issues, and practices spanning the K-8 program, one for each grade level K through 8, and one for an enriched eighth grade.

This set of monographs is intended as a **professional resource** for educators with responsibility for all or part of the elementary and middle-grade mathematics curriculum. Goals and outcomes described in these monographs are not meant to be prescriptive for local districts. Each district is expected to develop its own course of study for these grade levels. However, course description sections of the monographs can be a means of comparing and contrasting local curriculum efforts with these descriptions written for the whole state.

We encourage the reader of any monograph in this series to take the time to study other monographs. The introductory monograph *Teaching Mathematics in the Elementary and Middle Grades* provides a valuable total-program perspective with which to consider outcomes and practices at a given grade. Teachers of a particular grade level may gain additional insights into how to manage their part of the program by better understanding the program objectives and teaching techniques for the grades on either side of their grade. Curriculum directors and principals might wish to study all of the monographs for the grades under their supervision.

These monographs have been authored by a team of classroom teachers and university educators who believe that mathematics education can be improved through better communication and a shared purpose in educating children. We hope that these documents are helpful in your personal efforts to make your instruction more effective and rewarding.

SE 050 908

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
✓ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
B. F. Walter
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

BEST COPY AVAILABLE

INTRODUCTION

Children are the living messages we send to a time that we will not see.

John W. Whitehead

Kindergarten is a time for exploration, discovery, and individual growth. Unlike later grades when the sequence and pace of learning activities is fairly uniform, kindergarten is a level at which learners let the teacher know what is possible, how far an activity can go, how fast to move ahead, and *when to stop*. The role of the teacher is to interact with children individually, guiding and leading them through the early developmental stages of concept-building.

Developing ideas with young children is a fragile activity that cannot be hurried. A good analogy is the growing of mushrooms. Mushrooms cannot be willed to grow. But in a nurturing environment, in their own time, mushrooms spring up — often in clusters, though not always when or where we expect. There is a similar unpredictable quality to building ideas with young children. Children often view the same learning situation differently. They are influenced strongly by their prior experiences and intuition. They are limited in what they can accomplish by their level of cognitive development.

The challenge to the kindergarten teacher is to manage a classroom characterized by the variability and unbridled energy of its occupants. Within this setting, the teacher must acclimate children to the planned structure of group learning and still meet individual needs. Children need to feel comfortable in moving from their natural, play-style of learning to learning activities coordinated by the teacher. They still need to feel the freedom to explore ideas and raise questions of their own. They need to recognize that ideas "talked about" in class are relevant to their daily needs and activities. And they require frequent reassurances from a loving, caring adult.

In kindergarten, children have the opportunity to explore and develop ideas that are fundamental to the study of mathematics: number, counting, ordering, comparing, classifying, patterning, shape, size, position, numeration, measuring, adding, subtracting, and problem solving. Later instruction will formalize many of these ideas and be concerned with *measurable outcomes*. But for this year, the emphasis is upon experience and individual growth.

OVERVIEW OF THE KINDERGARTEN MATHEMATICS PROGRAM

In prenumber activities, children apply the processes used in play, **classifying, organizing, matching, and sorting sets.** Through these activities they develop the understanding that leads to the concept of number and the eventual operations of addition and subtraction. The emphasis in these activities is upon **developing and using language in manipulating physical materials.** Children learn to recognize and apply the attributes of physical objects in classifying, sorting, and patterning experiences. These activities prepare them for recognizing number as an attribute of a collection, for applying definitions in geometry, and for making generalizations from their observations.

Children extend their prekindergarten **counting abilities to counting objects through 30, counting-on, and counting-back.** They compare and order sets by matching and by number, and learn to recognize *one more/one less and two more/two less*. Activities involving concrete and pictorial materials are used in **assigning numerals to sets of objects up to 9.** Children learn to associate "how many" for a group of physical objects to the oral name for this number and the written numeral for this amount.

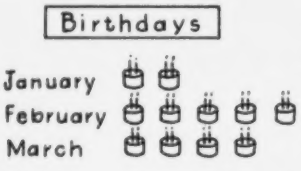
Many kindergartners attain **mastery in reading and writing one-digit numerals.** Through counting of sets, children are introduced to ordinal numbers through *ninth*. In preparation for addition and subtraction facts in first grade, children learn to regroup physical materials, naming the number in each subset, and to **use joining and separating to answer oral questions.**

Direct comparison and counting experiences are emphasized in measurement activities. Children **compare and order lengths, weights, and capacities** of physical objects. They are introduced to representing lengths with physical materials, such as strings (chain links) or Unifix™-type cubes, to make indirect comparisions. Vocabulary is developed for describing measurement

attributes and making measurement comparisons: *larger, smaller, heavier, lighter, more, less, taller, shorter, before, after.* Children learn to order events based upon time and recognize the clock and the calendar as means to measure the passage of time. They **recognize the value of the penny, the nickel, and the dime** and count coin values up to ten cents.

Vocabulary development and shape recognition are the focuses of geometry activities. Kindergartners learn to **identify spatial relationships,** such as *inside/outside, front/back, top/bottom, over/under.* They can also **recognize, describe, and name two-dimensional shapes:** square, rectangle, circle, and triangle. Through physical activities, learners sort and classify two- and three-dimensional materials by attributes (those with a square face) and by properties (those that can roll).

Problem solving is an informal kindergarten activity. Children use *strategies* and *manipulative materials* to answer oral questions. Fractions and graphs are introductory topics. Children learn to **recognize halves** of circular and square shapes, with an emphasis upon division into *equal parts.* Simple bar graphs are introduced by having children use stickers to create bars in answer to comparison questions.



KINDERGARTEN TEACHING CONSIDERATIONS

Kindergarten is the beginning of structured education for most children. Teachers face unique challenges in making this beginning a success for each learner. Among these challenges are these considerations in working with five-year-old youngsters: *variability, learning characteristics, language development, manipulatives, and personalizing instruction.*

Variability

Children arrive at kindergarten with a wealth of practical experiences and skills, but not with uniform amounts. Many have a sizeable working vocabulary of terms used in kindergarten mathematics. Some have attended preschool and have experienced prenumber, counting, and classification activities. Others have acquired mathematical ideas from viewing programs like *Sesame Street* or through experiences with their parents. But some have had very limited opportunities to shape mathematical ideas.

Many factors in children's environments influence their preschool knowledge (e.g., Knowledge of mathematical ideas appears to increase according to the age, intelligence, and socioeconomic level of the parents)[1]. Children's stage of development imposes limitations on what they are ready to learn. For example, until children can conserve number, they will experience difficulty with counting, addition, and subtraction. Furthermore, some skills of children may mask capabilities or weaknesses they possess that can affect their learning (e.g., Language fluency is not necessarily indicative of a child's ability to understand mathematical ideas).

Assessment is particularly important since **children do not begin kindergarten instruction with equal amounts of mathematics knowledge and experience.** Teachers need to determine early what children know and what they are ready to learn. Assessment also needs to be ongoing. Initial appraisals may reflect only a child's lack of prior experience or nurturing. This is particularly true for children who have been culturally malnourished. Given the opportunity, many youngsters can develop quite rapidly in spite of their background.

Although children vary widely in many areas of ability, studies do point out some broad generalizations about entering kindergarten students. Knowledge of these findings, coupled with individual assessments, can give the teacher a broad perspective with which to plan a range of activities for a given class.

Skills of Beginning Kindergarten Children[2]

- Many children can count and find the number of objects to 10; some are able to count to 20 or beyond.

[1]Suydam, Marilyn, and Weaver, Fred. "Research on Mathematics Learning," *Mathematics Learning in Early Childhood*, p. 49, National Council of Teachers of Mathematics 37th Yearbook, The Council, 1975, Reston, VA.

[2]Suydam, Marilyn, and Weaver, Fred. "Research on Mathematics Learning," *Mathematics Learning in Early Childhood*, p. 49, National Council of Teachers of Mathematics 37th Yearbook, The Council, 1975, Reston, VA and Rea, Robert E., and Reys, Robert E. "Competencies of Entering Kindergartners in Geometry, Number, Money, and Measurement." *School Science and Mathematics*, 71 (May 1971):393-94.

- Some children can say the number names for tens in order, but far fewer can say the names when counting by twos and fives.
- Most children know the meaning of *first*, and many can identify ordinal positions through *fifth*.
- Many children can recognize the numerals from 1 to 10, and some can write them.
- Most children can give correct answers to simple addition and subtraction combinations presented verbally with or without manipulative materials.
- Most children have some knowledge about coins, time, and other measures; about simple fractional concepts; and about geometric shapes.
- Most chidren are able to compare small groups of objects accurately.

Learning Characteristics

Kindergarten children are active, enthusiastic learners. They are curious and eager to explore new toys, new materials, and new problem situations. A kindergarten child works hard at pleasing the teacher, needs to experience success in class activities, and needs to be recognized for that success. Kindergarten children are not always convinced that something cannot be done by just observing another child — they often need to try it themselves. At five years of age, children are concrete learners and need ready access to physical materials at all times.

The variability that characterizes kindergartners' prior knowledge extends also to their learning characteristics. Motivation, anxiety, rate of response, curiosity, and impulsiveness vary considerably in their performance of learning tasks. Teachers can make activities more effective by noting these characteristics and by helping children to capitalize on their strengths. General tendencies possessed by most five-year- olds may also influence planning for instruction.

- They are largely dependent upon their perceptions and their personal experience in reaching conclusions.
- They are easily distracted by irrelevant aspects of a situation.
- They can usually hold only one thought or idea in their minds at a time.
- They are unable to discern contradictions in the logic they use.
- They cannot easily correct themselves when they make a mistake.
- They can often operate with concepts at a higher level than they can verbalize.

These generalizations are helpful in limiting expectations to the capabilities that kindergartners possess. Yet, teachers must also guard against underestimating the power of the processes that kindergartners use. The skills that kindergartners are developing enable them to engage significant questions. This is particularly true in problem solving.

> **"A remarkable discovery I've made is that my kindergarten students are successful problem solvers! Although their mathematical background is still undeveloped, their reasoning and insight are astounding."**
>
> Kindergarten Teacher

By using the skills at their command — such as counting, matching, forming groups — kindergarten children can perform quite successfully with oral problems. With little or no guidance, they can use manipulatives to solve problems involving all four mathematical operations prior to any formal arithmetic instruction.[3]

Put eight tiles in front of you. If you give me three tiles, how many will you have left?

Considering the learning characteristics of young children, **there is no urgency to move rapidly toward formal arithmetic instruction.** Children need ample time to build skills such as classifying, ordering, matching, and grouping, which underlie many later ideas. They

[3]Driscoll, Mark J. "The Role of Manipulatives in Elementary School Mathematics," *Research within Reach, Elementary School Mathematics*, p. 22, R & D Interpretation Service, CEMREL, Inc. St. Louis, MO.

also need the opportunity to wield these skills in meaningful activities that draw upon the thinking levels of which they are capable.

Language Development

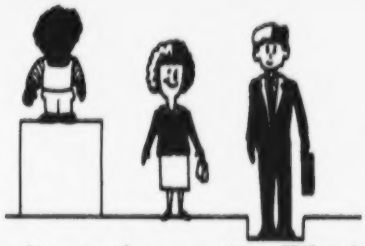
There are three key elements to language development in the kindergarten program: (1) building of a mathematics vocabulary, (2) using language to communicate ideas, and (3) using language to communicate understanding.

Language is often the means for communicating and extending ideas in learning. For example, studies show that kindergarten children use more than 200 words associated with mathematics, comprising as much as 6 percent of their working vocabulary. It is important for teachers to regularly expose children to the use of mathematics terms and to help them form accurate generalizations of what these terms mean. Children should see multiple examples of terms to avoid incorrect sets. They should also see nonexamples.

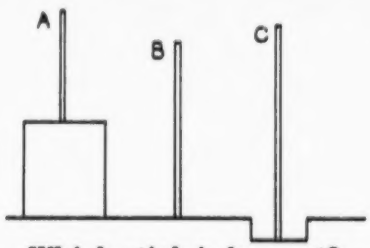
These are triangles. So are these. These are not triangles.

To communicate ideas effectively through language, terms must mean the same thing to both the speaker and the listener. Since language is often ambiguous, teachers need to be careful in giving directions and asking questions. Terms may not hold the same meaning for children as they do for the teacher. For example, a teacher might ask a class, ***Which of these shapes is the same?***

Young children often point to the circle and the ellipse. To them, these shapes are the "same" in the sense of being *rounded,* though the teacher means "same" as *identical.* Similarly, teachers need to be alert to the concept conveyed to children by the context in which a particular word is used.



Responding to the question, ***Who is tallest?*** (in the illustration above), children may choose the child standing on the box. They expect to make this kind of comparison by determining whose head stands highest rather than by using length as intended. They may also be distracted by irrelevant information, such as "Dads are always tallest." A similar question involving sticks rather than people might prompt a correct response, particularly if children have had earlier opportunites to make direct comparisons with sticks.

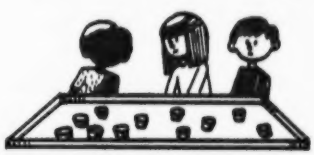


Which stick is longest?

By using examples to illustrate meaning, teachers can help children decide how terms are being used in discussion.

Language is only one way of communicating understanding. Since children are at different levels in their ability to express ideas, language should not be the only teaching means to determine understanding. Many children can understand and use mathematics concepts without knowing or using words to describe what they have done. Performance tasks permit children to demonstrate their understanding without the need for language.

Tami, show us on the flannel board how you answered this question at your desk, "How many marshmallows are left if three children share 11 marshmallows equally?"



Tami may be able to count out 11 marshmallows (pictures) and distribute them equally among

pictures of three children, but may not be able to explain what she did.

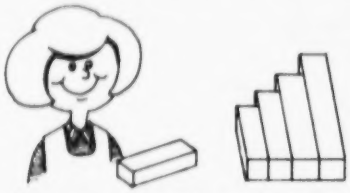
Other children may be proficient in using mathematical language but may not yet fully grasp the ideas associated with the words. Here again, performance tasks will disclose whether they have partial or full understanding.

Manipulatives

Since young children are so dependent upon their perceptions, manipulatives play a strategic role in their thinking. By being able to see, touch, and move objects, children can bring several senses into play in a single activity. In kindergarten, **children should be introduced to and have available a variety of manipulative materials.**

Learning should regularly involve manipulative materials as mathematical ideas are explored. Children need frequent reinforcement with manipulative activities and ample time for exploration. Teachers can use small-group discussions to introduce manipulative activities and to evaluate and record children's peformance.

Research consistently points out that using manipulatives is valuable in raising achievement and that the teacher needs to be a central figure in guiding children in their use. Besides using questions to direct a manipulative activity, teachers should also ask questions to get feedback on children's thinking in their use of manipulatives.



Josie, what are we using the Cuisenaire™ rods to show?

Chris, why did you move the tiles that way?

Kindergarten children need numerous **activities with manipulatives that require them to *count, join, separate, group, compare,* and *distribute.*** These are the key processes behind children's thinking at this age. These manipulative experiences build the prerequisite ideas for numeration, addition, subtraction, multiplication, and division work with symbols.

Following are seven types of activities in which kindergartners use manipulatives.

- **Counting** — Children need numerous opportunities to develop counting strategies in answer to questions — point-counting, counting-on, and counting-back (see *Counting* on page 11).

- **Numeration** — In preparation for place-value in first grade, children need to group into tens and ones, and regroup to ones and count.

- **Part-whole grouping** — Children need experience in seeing the various ways that a set can be decomposed into subsets as preparation for addition and subtraction (e.g., 6 cubes can be regrouped as 5 and 1, 4 and 2, 3 and 3, three 2's).

- **Money** — Children often demonstrate a great range of proficiency with coins, some requiring numerous experiences with trading, counting-on and "buying" to become familiar with coin values.

- **Geometry** — Hands-on materials give learners the means to classify objects by characteristics and to note similarities and differences in the properties of objects.

Point to objects in this group that can roll.

Hold up these objects:

(a) something that *always* rolls in a circle.

(b) something that *always* rolls in a straight line.

(c) something that *changes directions* if it bumps into something while rolling.

- **Measurement** — Children use comparing and counting in kindergarten measurement. They use parts of their bodies to determine how long something is or how much longer one object is than another. They use other objects in making measurement comparisons (e.g., weighing, pouring, covering an area).

- **Guess and verify** — Questions with answers that can be verified by manipulatives begin children's introduction to estimation and the use of visualization skills. Children enjoy the challenge of guessing and the use of their counting skills to check how well they did.

  If you cover your card with chips, guess whether it will take more than 10 chips or less than 10 chips. On your card, make an "M" for more than 10 chips or an "L" for less than 10 chips.

Children can sometimes learn by observing others using manipulatives. They frequently learn to play games by watching others. They might observe classmates working with a magnetic counter board or the teacher using manipulatives as an illustration.

In using manipulatives, teachers need to be mindful that children are easily distracted by irrelevant details. Their attention must be kept focused on the attribute being considered. Although children do not generally experience much difficulty in allowing one object to represent another, questioning should still be used to assure that misinterpretations do not occur.

Manipulatives should be simple and should suit the task. Materials do not need to be sophisticated or expensive, but they do need to be available in wide variety and in sufficient quantity to let all children participate in exploratory activities. Concrete materials should be supplemented with pictorial materials to permit explorations to continue with paper-and-pencil, seat-work activities and to build bridges from concrete to semi-concrete understandings.

Some activities and concepts are best explored through continuous materials like counting rods and others through discontinous materials like trading chips. Some children may show a natural preference for one type of material over the other. But it is important for children to experience and use a variety of manipulatives to illustrate the same concept. Variety provides for the multiple embodiment of an idea, so important in generalizing a concept. Later in the year, children can be given a certain amount of freedom in choosing manipulatives to help them answer questions.

Personalization of Instruction

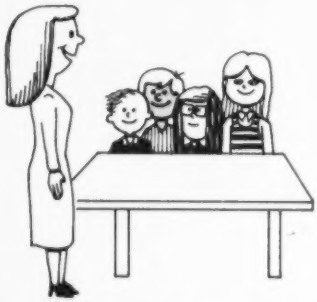
Studies on the effectiveness of kindergarten instruction consistently point to the importance of teacher-child interactions. Learning depends upon how well the teacher can connect old ideas with new in guiding children. Whether a child can recognize the similarity of ideas between models or make a connection between examples depends upon how well the teacher can make those ideas clear and explicit for that child.

Frequent praise and encouragement are vital to motivate young children and keep them on task. They work in large part to please an adult. They seek frequent approval for their work and need to feel that the teacher has a personal interest in them.

For each child to experience success, the teacher must keep that child's individual development uppermost in mind in interactions. Some children will be verbally fluent and readily respond to questions. Others will not be able to explain easily but can easily accomplish performance tasks. Children with rich prior learning experiences may be markedly advanced in some areas, as compared to their less experienced classmates. But **children from less supportive backgrounds can usually advance quite successfully when given appropriate developmental experiences.**

Teachers will eventually want to use both large-group and small-group instruction for mathematics. Small-group work allows more individual interaction. It is more effective for introducing new ideas and working with children who are at a less mature level: less secure, less verbal, less confident. Small groups provide the teacher with greater opportunities to assess student understanding, adjust learning tasks for an individual's level of understanding, and engage children in meaningful language practice.

Large-group instruction, a target toward the end of the year, provides the opportunity to promote socialization skills, such as communication, cooperation, responsibility, and mutual respect. But large-group work requires children to be comfortable with school, able to participate in discussions, and disciplined enough to stay on task with minimal supervision. In large-group work, differentiated questioning must be used to match expectations to a child's experience, motivation, and ability level. Tasks must be selected with care to assure that each learner can experience a measure of success and be an active participant in the large-group activities.



Great flexibility should be exercised in forming and using learning groups. Besides learners' inherent variability due to incoming differences, children are at different degrees of readiness for different areas of study. For example, a child who can conserve number may not yet be able to conserve length. It is especially important that children be allowed to advance at their own pace. Therefore, learning groups need to be reformed frequently to match children's needs.

Though kindergartners possess many capabilities and learn rapidly, there should be no great rush to move them toward a "first grade program." It is quite easy to move beyond their level of comprehension. A good kindergarten program should place the **major emphasis on giving learners experiences in applying and extending the processes that they bring with them to this level.** It should also help them to develop the language to express related ideas.

> The focus of a beginning program should be (1) accumulating sensory information, (2) organizing that information into concepts, (3) acquiring appropriate language, and (4) developing the intuitive background for future symbolization.[4]

[4]Gibb, Glenadine, and Castenada, Alberta. "Experiences for Young Children," *Mathematics Learning in Early Childhood*, p. 97, National Council of Teachers of Mathematics 37th Yearbook, The Council, 1975, Reston, VA.

TEACHING EMPHASES

This section discusses content strands that play an important role in the kindergarten mathematics program: *Prenumber Experiences, Number Activities, Measurement, Geometry,* and *Problem Solving.* Discussions are illustrated with sample questions and activities.

Prenumber Experiences

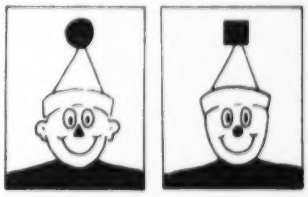
The concept of number is a significant abstraction requiring careful development in kindergarten. There are several important processes that set the stage for the meaning of number: *describing sets, classifying sets, sorting sets, matching sets, comparing sets,* and *ordering sets.* These are processes that children have used naturally in play prior to school. They undergird not only the concept of number but also many other important ideas and activities in mathematics.

Describing Sets

Kindergartners need many experiences with describing objects and pictures. These activities give them the opportunity to develop and use vocabulary for attributes of objects and figures. They also gain practice in discerning distinguishing characteristics. Number is one way to describe an attribute of a collection of objects.

As a whole group activity, ask for as many words as possible to describe a puppet, a clown picture, a toy, or an animal. After several open-ended activities of this type, children will be ready for a more structured activity, such as the following:

Tell one thing that is different about the two clown pictures.

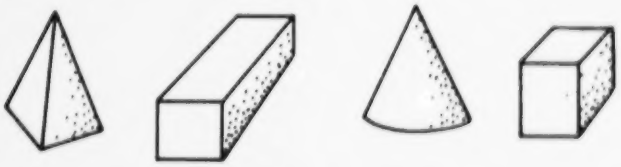


Choose three words to describe one of the solids in front of you so that your partner can pick it out.

Classifying and Sorting

Classifying and sorting require children to apply a rule in making decisions about the members of collections (e.g., blocks, toys, pictures). The simplest rule is to find objects identical to the one given. Children then progress to use of one attribute in applying a classification or sorting rule to similar objects. Next, children may form two or more groups according to their own rules. Finally, children are asked to identify the attribute used to form one or more groups.

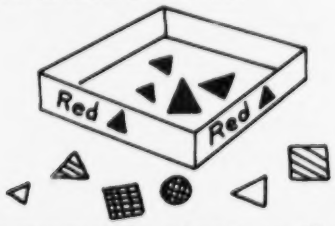
Elisha, sort these geometric solids so that those with any square faces go in this box and those with no square faces go in the other box.



Carrie has sorted these things into two piles. Can anyone guess the rule she used?

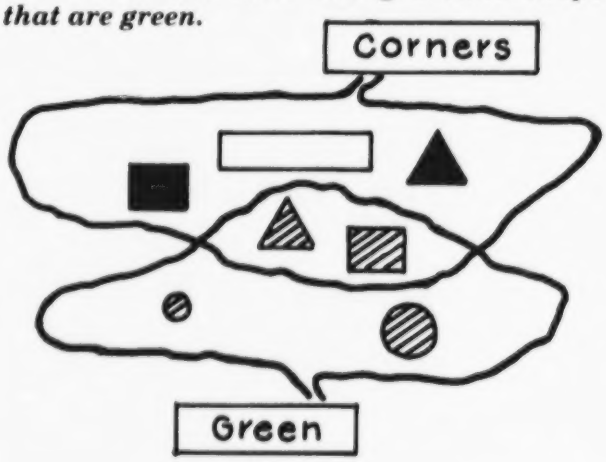
Classifying and sorting experiences are important in preparing learners for the concept of number. A number is an attribute of a collection when it is considered as a whole. In recognizing the number of objects within a group, the child must disregard similarities or differences among the objects to focus exclusively upon the *numerosity* of the group.

After several activities with describing, classifying, and sorting by one attribute, some children can be challenged with similar activities involving two or more attributes. A collection may also contain some objects that belong in more than one group when sorting is applied. These activities should be considered enrichment since many kindergarten children have difficulty considering more than one idea at a time.



Sort the geometric pieces so that ones that are triangles and <u>also</u> red go in the box. Let the rest stay on the table.

Separate these shapes into two groups. Place one string around shapes with corners. Place the other string around shapes that are green.



Repeating and Extending Patterns

Repeating or extending a pattern also requires children to recognize and apply a rule. Patterns can be demonstrated with colors, geometric shapes, sizes, numbers, textures, and even sound (in rhythmic activities). At the introductory level, children play with manipulative materials and develop their own patterns. Teachers can encourage them to continue a pattern they have started and then ask another child to extend that pattern.

Copy this pattern with attribute shapes.

Keep this pattern going by adding more pieces.

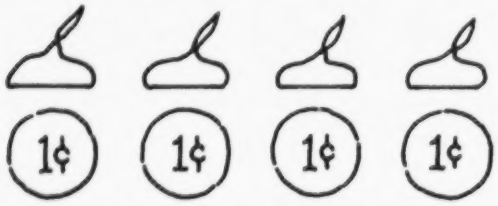
Other materials useful in patterning activities are buttons, beads, colored macaroni, and different macaroni shapes.

Matching, Comparing, and Ordering

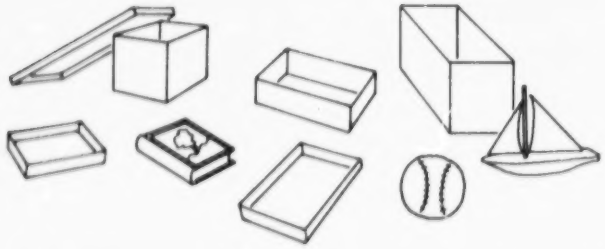
Children have an intuitive understanding for small numbers that are used to tell how many of a group (e.g., 3 doughnuts). *Matching, comparing,* and *ordering sets* enable children to build an idea of number as an abstraction — an attribute that different sets can possess in common. Numerous

experiences that **match sets one-to-one** are necessary to establish (1) the notion of equivalence or nonequivalence between sets of dissimilar objects, and (2) the "fourness", for example, that two sets of four objects have in common.

Physically establish a matching by placing one set of objects one-for-one on another set. Show children that the coins match the candies and the candies match the coins.



Each toy has a box and each box has a lid. Have children discuss all possible matchings shown by the picture (e.g., toys-boxes, boxes-lids, toys-lids).

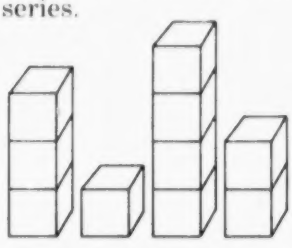


Children need to experience these ideas with many concrete manipulatives as well as with pictures. Teachers should also take advantage of incidental opportunities for one-to-one matching. For example, have a child from one learning group place a chair at the table for each person, get a worksheet for each chair, and then put a pencil with each worksheet. Then ask, ***How do we know that each child in the tiger group has a pencil?***

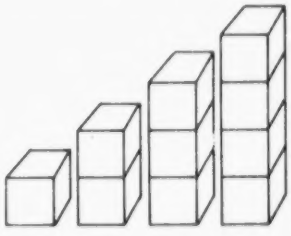
In comparing nonequivalent sets, students begin to use the terminology *more, less,* and *the same* to distinguish among comparison situations involving the attribute *number.* Through matching, learners can establish that one set has *one more (less)* or *two more (less)* than another without counting. These concrete experiences can later be linked with the notion of counting-on in enumerating sets.

On your papers, use matching to compare the balls to the gloves. Which is more? How much more? Which is less? How much less?

Activities with one-to-one matching lead naturally to the idea of **ordering**. Ordering activities which involve number should emphasize *one more* and should ask children to arrange existing sets in order or to construct the next one in a series.



Arrange these towers of connecting cubes in order.



Add the next tower to this staircase.

Number Activities

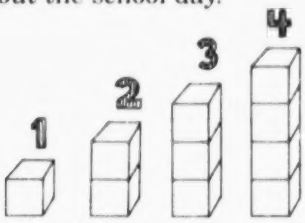
Kindergarten children are expected to learn a great deal about number and number names as well as to begin introductory experiences with whole-number addition and subtraction. Many children will enter kindergarten with a number vocabulary to ten (or more) and the ability to rote count to at least ten. But they may not have developed very useful insights into what a number signifies. Teachers must take precautions to insure that entering skills and those to be developed are based on an extensive background of understanding through manipulative experiences. **Concern for building a strong background of meaning and experience should outweigh concern for skill development at this level.**

These six major ideas concern the development of the concept of number in kindergarten:

- Number as an attribute of a collection of similar (or dissimilar) objects
- Number as a means for comparing two sets
- Number as a means for ordering sets
- Symbols (numerals) to tell how many in a set
- Relationship of number of elements in a set to counting
- Ordinal number to describe placement within an arrangement or time sequence

Numeration

Assigning numerals to sets of objects is related to rote counting and to arranging sets in order by number. Kindergarten children need many opportunities to practice using numerals to tell the number of objects in a collection. These activities should be done in order originally and later revisited randomly through planned lessons. Teachers can simultaneously take advantage of the many incidental counting and number situations that arise throughout the school day.



Children at this level are expected to learn to read and write numerals up to nine. Reading and writing activities should be preceded by numerous experiences involving the naming of *how many* for sets of concrete objects. Numeral writing will be difficult for some youngsters. Use of cutout, sandpaper, or prepared plastic numerals can be quite effective before and during the time that writing skills are developing. Tracing cutout numerals is also helpful.

Counting

Counting is a key activity in five-year-olds' strategies for solving problems and answering questions. They rely upon a variety of techniques and have generally progressed or are progressing through the following stages:[5]

- **Rote-counting** — Recites number names in sequence without being able to match names to objects
- **Point-counting** — Counts objects by pointing to each in turn while reciting a number sequence, but fails to recognize the total number for the set "counted"
- **Rational-counting** — Tells how many in the whole set after counting
- **Reproductive-counting** — Goes to a large collection and gets a specified number of objects
- **Counting-on** — Begins a counting sequence by starting with a known number of objects and counts on to find the total
- **Counting-back** — Starts with a known number of objects and counts backward a specified amount
- **Counting-on or counting-back mentally**– Counts on or back a specified amount (e.g., back 3 from 15) without resorting to objects

These counting techniques prepare children for (1) efficiently answering joining and separating questions, (2) regrouping a set, and (3) counting money and determining time. Related subskills used in counting strategies are: finding one more/one less, finding two more/two less, and learning to recognize groups of objects from 0 to 5 without counting.

Children should have both formal and informal opportunities to count throughout the day. Many counting opportunities occur naturally if children participate in passing out and collecting materials. To motivate the need to count-on or count-back, teachers might conduct activities that illustrate how these strategies are more efficient for telling "how many" than rational-counting.

Put a few beans in a small bottle and tell the class that amount. Ask a child to add two more beans to the bottle and then tell <u>how many</u> are now in the bottle.

Next, ask another child to remove a bean and tell <u>how many are left</u> in the bottle.

Initially, it is a good idea to verify counting-on and counting-back results (e.g., empty the bottle of beans onto an overhead projector and use point-counting) until children are convinced that they will get the same result.

Ordinal Number

Many children enter kindergarten with an understanding of *first*. But far fewer understand the meaning of *second* and very few the meaning

[5]Driscoll, Mark J., "Counting Strategies," *Research within Reach, Elementary School Mathematics*, p. 18, R & D Interpretation Service, CEMREL, Inc., St. Louis, MO.

of third. In kindergarten, children learn the ordinal number names through *ninth*. Children can learn that ordinal numbers are used when things are "lined up" or when "one happens after another."

Ordinal ideas can easily be associated with counting by having children line up to go out the door. Have each child remember her number as you count those in line, beginning nearest the door. Next, count the children with ordinal numbers, ***first, second, third, fourth, fifth ...*** Then ask children to hold up their hand in response to these questions:

Who will go out the door first? Who is number one?
Who is number five? Who is fifth?
Who is fourth? What is your number?
Who is number two? What is your place in line?
Who is last? How many children are in line?

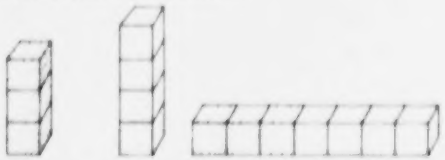
Children can quickly associate ordinal-number names with a numbered position when counting. They have only to learn the special names for first, second, and third. Kindergartners can also learn to use ordinal numbers to indicate the sequence in which things happen. Cut out an interesting cartoon strip, cut into individual frames, and mix the frames.

Choose the picture that shows what happened <u>first</u>.
Choose what happened <u>next</u>. What happened <u>next</u>?
Now point to what happened <u>second</u>; <u>last</u>.

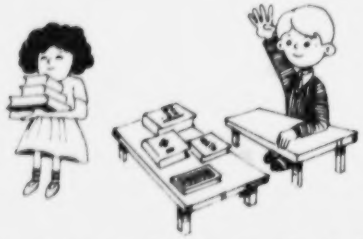
Addition and Subtraction

Introduction of informal addition and subtraction occurs through the use of concrete manipulative experiences. Children learn to model addition and subtraction situations with sums up to nine — using cubes, sticks, checkers, and other physical objects. Little or no emphasis is placed on developing speed with addition and subtraction facts. Addition and subtraction are normally developed together so that learners can begin to recognize the relationships between the two operations.

Children themselves make excellent physical models as they act out number stories. Pictures and diagrams are also used to help solve addition and subtraction situations.

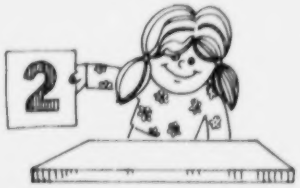


Build a tower with three red blocks. Now build a tower with four blue blocks. Put the two towers together. How many green blocks should you ask for to build a tower the same height?

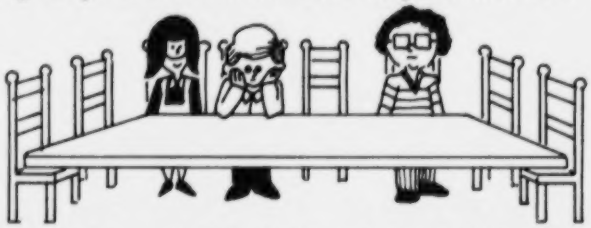


There were seven books on the table. Three books were taken away. How many remain?

Gina, will you come up to the table and act this out? While Gina is doing this, I want the rest of you to show with your fingers <u>on your desktop</u> what you think the answer will be.



There were five books on the table. Now there are three books left. How many were taken away? Choose a number card that gives the answer. ... Now pretend that counting chips are books and check your answer.



There are eight chairs at the table. Three children go to the table and sit down. How many more children can go to the table and sit down?

13

Measurement

Kindergarten measurement explores the foundations of measuring activities. The purpose of these activities is not mastery but to make the notion of comparing particular common attributes meaningful. **Children are at the perception and comparison stages of measurement development,** sometimes referred to as premeasurement. They are not yet ready to consider measuring in terms of nonstandard or standard units. For example, asking them why the room is a different number of "steps" wide for Jimmy than for Ann can be confusing.

The teacher's first concern is to be sure that children are considering the attribute which is the focus of the measurement activity. This is not always clear; most objects have more than one obvious attribute in common. For example, in comparing the capacity of boxes, children can easily be distracted by height, width, length, or shape. Comparison activities should make it easy for children to discern the attribute being discussed (e.g., boxes that can be nested to make the capacity comparision clear.)

Another problem is terminology. Does the term used to refer to the attribute being compared mean the same thing to the child as to the teacher?

Most children agree that two broomsticks placed next to one another are the same length.

But if one is advanced ahead of the other, some children now believe one is longer. Apparently, "longness" has something to do with children's spatial perception of the broomsticks.

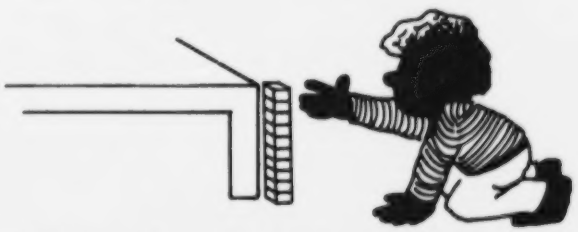
Learners need many experiences in **comparing and ordering objects that can be compared directly.** Direct comparison focuses children's attention more squarely on the attribute being compared. It also gives children the perceptual experience to associate with vocabulary used in the comparison.

Who is taller?

Stand in order of height.

Which weighs more?

Perception should be the means for making measurement comparisons, even those involving indirect comparison. For example, two tables may be too heavy to compare heights directly. But children can build a tower of connecting cubes the *same height* as one table and carry this tower over to a wall chart to make a mark to show that table's height. Repeating this process with the other table, they can make a second mark and then compare these marks to heights on their body. Some children will clearly see more efficient ways to do this, but they will benefit by this experience, which appeals directly to their senses.



Number and nonstandard units often enter into a comparison situation. But rather than using numbers indirectly for comparison, activities should be designed in such a way to give students direct experience in perceiving the attribute being compared.

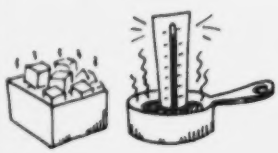
Ask children which of two pitchers they think holds more punch. Then have one child fill as many glasses as possible from one pitcher, placing those glasses in front of the empty pitcher. Have another child repeat with the other pitcher.

Now ask the class which "full" pitcher they would rather have; why; and how they can explain their choice.

In kindergarten, learners begin to recognize that some instruments are used to make measurement determinations. Balances can compare weights of objects when *hefting* does not give a clear answer. The clock and calendar use numbers to indicate the passage or the duration of time. A thermometer shows the temperature. Here again, the emphasis should be upon appealing to a child's senses rather than to number.

Is a Yo-yo heavier than one block? two? three?

Let children see the rise of the mercury on a thermometer when transferring it from ice water (which they feel) to warm water (which they feel).



The kindergarten child's sense will be at a primitive experience level for many measurement ideas — time is perceived as *before now, now,* and *not yet*. Before children are ready to quantify attributes or see how one attribute is related to others, they must be able to focus singly upon a common characteristic possessed by two objects and decide which has more and which has less of that attribute.

Geometry

By age five, most children are able to accomplish the following geometric objectives:

- Distinguish, though not necessarily name, circles, squares, and triangles from one another
- Correctly reproduce recognizable copies of these figures
- Demonstrate their understanding of these figures as simple, closed curves
- Draw line segments in the correct orientation (vertical, horizontal, oblique) as part of a figure[6]

To these abilities, the kindergarten program adds experiences in *describing, classifying, constructing,* and *exploring*. Children begin their study of geometry by describing and classifying geometric shapes. Two-dimensional shapes are sorted by number of sides or number of corners (angles). Children learn to name triangles, squares, rectangles, and circles. Simple three-dimensional (space) figures are sorted by number of faces, number of edges, and characteristics such as "which will roll" or "which will stack."

Informal discussion of closed and open paths is used to help children learn spatial relations. For example, a walk that returns to the starting point represents a closed path and can lead to examples of inside and outside. Other spatial relations, such as *top, bottom, above, below, before, after,* and *between,* need to be acted out and pictured.

Activities involving physical materials continue to be effective means of involving children's perceptions in concept-building. For example, attribute shapes and blocks allow the teacher to direct the child in tasks requiring discrimination, classifying, and sorting of two- and three-dimensional figures.

Can you find a flat shape with four corners?

Pick out a plastic piece that looks like a "car." Can you make it look like the "car" on the first card? the second card?

Find shapes that look like this.

How are they different?
How are they the same?

Children particularly need activities that require them to construct something. These tasks focus their attention on the attribute required in the construction.

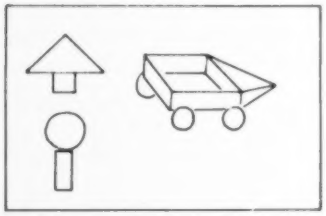
Mavis will hold the pipe cleaners. Choose a shape card. Decide how many pipe cleaners you need to make the shape. Ask Mavis for that many. Then make the shape.

A construction activity can also make children aware of how more complicated figures are made up of simpler parts. Such activities help children recognize the use of geometric ideas in their environment.

[6]Robinson, Edith. "Geometry," *Mathematics Learning in Early Childhood*, pp. 207-8, National Council of Teachers of Mathematics 37th Yearbook, The Council, 1975, Reston, VA.

Choose two attribute blocks to make a tree. Name the blocks.

Use the attribute shapes to make a picture. ... As I point to something in your picture, tell me its name. Then tell the name of the shapes used to make it.



Geometry activities with physical materials have these advantages for children: (1) they can decide whether or not they have done the task successfully; (2) they can perceive a purpose behind the activity; (3) they are developing visualization skills — making complicated figures from simpler ones or "seeing" simpler figures in a more complex figure; and (4) they are broadening their generalizations by recognizing figures in different orientations, sizes, and contexts.

Problem Solving

Teachers need to establish a problem solving, exploration-centered classroom — an environment in which kindergarten children are expected to understand as well as to know. The program should include **numerous experiences with problem solving,** particularly situations that involve numbers and measurement. Children can begin to learn problem-solving strategies and how to use these strategies in interesting problem settings. The following strategies are introduced informally in kindergarten.

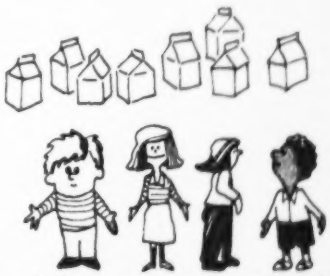
- **Act out the story or problem.**
- **Make a drawing or diagram.**
- **Guess and check.**
- **Make a model.**

Simple story problems that describe *joining groups, take away, comparison,* and *adding on* are good problems for this level. For this type of problem, the teacher might tell the story and then let children act out or model the situation described in the story to find the solution. After a solution has been found, it is a good idea to discuss the problem to see if the answer fits and to review how the problem was solved.

In the same way that a favorite story may be read over and over, so too can children profit from acting out the solution to a problem on several occasions. Children enjoy the repetition and the security of familiar situations. By selecting activities rich in exploratory potential, teachers can slowly extend these activities to include additional concepts as dictated by children's interest and comfort level.

The classroom can be a rich source of meaningful problems. Problems can be found in arranging the daily schedule, distributing supplies, or caring for classroom pets. Creative teachers will pose problems from all parts of the day.

We had four goldfish. Then one died. We just added two more goldfish to the tank. How many do we have now? Can you tell without looking?



We have eight containers of milk and four children. If the milk is shared equally, how many containers will each child get?

Most instructional activities for kindergartners have a problem-solving flavor when the activity appeals to the natural processes used by children at this age. Left to their own resources, children develop practical, informal, nonwritten routes for addressing mathematical tasks.[7] They begin to develop and apply their own thinking strategies. The key in their development is to **postpone premature symbolization** to allow them to exercise their natural problem-solving skills.

[7]Driscoll, Mark J., "Counting Strategies," *Research within Reach, Elementary School Mathematics*, p. 9, R & D Interpretation Service, CEMREL, Inc., St. Louis, MO.

All the children shown are leaving school to go to Chandra's party. Four children will ride in each car. How many cars are needed?

Mr. Meanie's goat has to be tied up to keep out of trouble. We have several pieces of rope to tie him to the post. He must be able to reach the water but not get into the petunia patch. Which rope should we use?

In addition to the major strands outlined above, kindergarten includes an introduction to other topics, such as *simple fractions, beginning time, simple money, the calendar,* and *graphs.* These topics should be spread throughout the year and used in problem-solving settings whenever possible.

MAJOR LEARNING OUTCOMES FOR KINDERGARTEN

Learning outcomes for kindergarten are presented under the strands of the elementary program. The following list of outcomes should not be interpreted as imposing limits on the kindergarten program. There are many other introductory activities and understandings that are part of this year's work. The following outcomes are the major competencies that children will be at or near mastery level by the end of this year.

This list includes both *process-related* and *product-related* outcomes. Those outcomes which relate to children's understanding of **concepts, properties, and processes** are identified in the list by this symbol •. Experienced teachers will recognize that these outcomes are often difficult to assess, but that they are *crucial* to learner understanding and long-term retention.

<u>Describing and Manipulating Sets</u>

- Describe objects or pictures according to distinct attributes
- Classify and sort objects or pictures according to two different attributes
- Recognize and demonstrate one-to-one matching of three sets with numbers less than 10
- Recognize and continue patterns with objects

<u>Numbers and Operations</u>

- Compare and order sets by number for sets of up to nine elements
- Read, write, and assign numerals to sets of up to nine elements
- Count by rote to 30
- Use ordinal numbers up to "ninth"
- Physically model addition situations with sums to nine
- Physically model subtraction situations with sums to nine

<u>Measurement</u>

- Compare and order lengths
- Compare and order weights
- Compare and order capacities
- Model the results of joining or separating lengths, weights, and capacities
- Order events based on time
- Count pennies and nickel to 10 cents

<u>Geometry</u>

- Recognize and describe spatial relationships, such as inside/outside, top/bottom, before/after, and over/under
- Recognize, name, and describe squares, rectangles, triangles, and circles
- Describe common three-dimensional shapes
- Use physical materials to construct simple geometric shapes
- Classify and sort geometric pieces

<u>Problem Solving</u>

- Demonstrate a willingness to begin a problem
- Demonstrate a concern that an answer fit the question
- Model or act out one-digit joining and separating stories
- Find one more/one less and two more/two less with objects
- Create one-digit joining and separating stories

Estimation

Make reasonable estimates for which is taller, longer

Make reasonable estimates for which is heavier

Fractions

Recognize and name halves of circles and squares

Graphing

Construct and read simple bar graphs

EVALUATION

The kindergarten program described in this monograph contains a wide range of content and emphasizes processes as much or more than it emphasizes skills and facts. Success with skills and facts is usually easy to measure with paper and pencil techniques. But those outcomes that stress meaning and understanding usually require other means of assessment. Teachers need to acquire and feel secure in relying upon a variety of evaluation techniques for mathematics learning.

The importance of measuring the full range of learning outcomes cannot be overstressed. Programs often emphasize those outcomes that are most easily measured. Consequently, outcomes which stress meaning and understanding frequently receive less attention than they should.

Kindergarten teachers become quite accomplished at observing children's social development and reading readiness. With planning, these same observation skills can be employed to assess many important outcomes of the mathematics program. A simple check-list can provide a record of learner progress toward the achievement of some outcomes, also serving to validate the observations. In using this technique, a teacher observes only a few children during a single lesson.

A similar checklist can be used to keep a record of interviews. Interviews are a powerful means of evaluating success in developing meaning and understanding. The teacher can ask follow-up questions as the child uses a model or draws a picture. With a little practice, a teacher can become a skilled user of the interview technique. Interview questions generally require an action by the child.

If we put two more blocks on the red tower, which will be taller — the red tower or the blue tower?

Use the counting chips to show the answer to this story ...

A FINAL WORD

The building blocks of mathematical understanding are **concepts** (e.g., *greater than, division by distributing equally, percent as part of a whole).* Concepts enable children to bring sense to the operations they perform. They provide a framework within which skills are learned and retained. They supply children with the insights necessary to apply and adapt what they know. Children who have a sound conceptual background are capable of **thinking and acting mathematically.** Children not well-grounded in concepts are forced to resort to memorization and superficial guesswork in confronting mathematical tasks.

In this respect, teaching mathematics is very much a team activity. Seldom are concepts fully developed during one year's instruction. Children progress through stages **(concrete → representative → abstract)** in acquiring a full understanding of a concept. Instruction at each successive grade level must build upon the ideas developed previously. Assessment of a child's *understanding* at any particular grade often reflects the culmination of the efforts of teachers over several grades in building a particular idea. Conversely, important instruction conducted by a teacher during the year may receive relatively little assessment emphasis that year, even though that instruction may eventually be crucial to whether children comprehend subsequent material.

For these reasons, communication between grade levels is essential to the sucess of the total program. Teachers must appreciate how their efforts contribute to the success of the program as a whole. This series of monographs attempts to show how the instructional development of ideas at a particular grade is related to that at other grades *as well as* to identify the fundamental outcomes associated with each grade.

We encourage teachers to study other monographs in this series to gain an even better appreciation of the teaching challenges of their particular teaching assignment. Hoping that you find this series helpful and informative, we invite you to send comments to the **Division of Inservice Education, Ohio Department of Education, 65 South Front Street, Columbus, OH 43266-0308.**

RESOURCES

The foregoing pages describe an activity-centered kindergarten classroom. Such a classroom requires many manipulatives and other supplies. Teachers will want to regularly add these materials to their collection of activities. The following list of resources is helplful in maintaining a collection of supplies and activities.

Mathematics Teaching in Early Childhood. 37th Yearbook of the National Council of Teachers of Mathematics. Reston, VA: The Council, 1975.

Arithmetic Teacher. Journal of the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22901. (Each issue contains ideas for activities designed for or adaptable to kindergarten.)

Research Within Reach, Elementary School Mathematics. R & D Interpretation Service, CEMREL, Inc., St. Louis, MO.

Baratta-Lorton, Mary. *Mathematics Their Way.* Rolling Meadows, IL: Addison-Wesley.

Creative Publications, Inc., 5005 West 110th St., Oak Lawn, IL 60453.

Delta Education, Box M, Nashua, NH 03061.

Dale Seymour Publications, P.O. Box 1088, Palo Alto, CA 94303.

Cuisenaire Company of America, 12 Church Street, Box D, New Rochelle, NY 10802.

STATE BOARD OF EDUCATION

Patricia Smith, Worthington, *President*
Paul Brickner, Willoughby, *Vice President*
Jean E. Bender, Akron
Richard C. Glowacki, Toledo
Mary Goodrich, Zanesville
Jack C. Hunter, Youngstown
Virginia E. Jacobs, Lima
Mary R. Lindner, Cincinnati
William E. Moore, Woodsfield
Sue Ann Norton, Westerville
Paul F. Pfeiffer, Massillon
Ronald M. Popek, Cleveland
C. J. Prentiss, Cleveland
Edwin C. Price, Jr., Cincinnati
Constance Rice, South Charleston
Chester A. Roush, Kettering
Anthony J. Russo, Mayfield Village
Wayne E. Shaffer, Bryan
Sally R. Southard, Oxford
Jo A. Thatcher, Portsmouth
Karen L. Worley, Elyria

OHIO DEPARTMENT OF EDUCATION

Franklin B. Walter
Superintendent of Public Instruction

G. Robert Bowers
Assistant Superintendent

Nancy A. Eberhart
Director, Division of Inservice Education

Principal Authors:
James B. Wesson
Professor, Miami University

Steven P. Meiring
Mathematics Specialist
Division of Inservice Education

Consulting Teacher:
Carole E. Listiak
Tremont Elementary School

Monograph Development Committee

Patricia D. Garner, Grade 2, Jefferson Elementary School, Parkersburg
Margaret B. Kasten, Department of Theory and Practice, The Ohio State University
Linda Kazee, Grade 8, Finland Middle School, South-Western City Schools
Carole E. Listiak, Grade 1, Tremont Elementary School, Upper Arlington City Schools
Rebecca E. Martin, Grade 7, Washington Junior High School, Washington Local Schools, Lucas County
Steven P. Meiring, Mathematics Specialist, Ohio Department of Education
Nancy K. Ross, Grade 7, Harding Middle School, Lakewood City Schools
James E. Schultz, Department of Mathematics, The Ohio State University
C. Winston Smith, Jr., College of Education, Ohio University
William R. Speer, College of Education, Bowling Green State University
Marilyn N. Suydam, Department of Theory and Practice, The Ohio State University
James B. Wesson, College of Education, Miami University

An Equal Opportunity Employer